OCEAN TIDES AND FRACTAL GEOMETRY: TIDAL STATION STABILITY

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ABSTRACT

Fractal Geometry is one of the most powerful tools in the research of complex dynamical systems of high, fractional or unknown dimensions. It allows the computation of the dimensions of attractors of the phase space of the dynamical system and their stability. The authors apply some fractal concepts on tidal elevation data sets considering them as the projection of the temporal evolution of the dynamic system of the tide in the vertical axis. At the same time the mean sea level is considered the attractor of the dynamic in such way that the system evolves around it. The three considered tidal stations are Ceuta, Malaga and Algeciras. In addition a theoretical standard from the tidal potential RATGP95 has been used for comparison.

INTRODUCTION

A fractal is a non usual geometrical object with a dimension greater than its topological one (Guzmán et al. 1993). They have been also named mathematical monsters (Pla 1994) because they present very different geometrical properties than usual geometrical sets. The vector and functional spaces of non integer or transitional dimensions are included in fractals. In 1919 Hausdorff built up the main tools to compute measures on that sets in the known as Hausdorff measures and dimensions. In the twenties, Besicovitch developed their geometrical properties (tangency, projections and densities) and he called them strange sets. His original and revolutionary ideas are the actual basis of the Geometrical Theory of the Measure. In the seventies B.B. Mandelbrot made many papers naming that sets as fractals, making popular the theory and pointing out its possibilities. A nice resume of fractals and its application on Geophysics can be found in Mandelbrot (1989). Fractals were applied on many fields of the Earth Science (i.e.): Geology (Turcotte 1989; Todoeschuck & Jensen 1989; García & Otárola 1994), Seismology (Crossley & Jensen 1989), Physical Oceanography (Provenzale et al. 1989). Nowadays the fractals are the central point of many mathematical researches with wide applications.
There are roughly three types of fractal sets: a) Common mathematical entities with the same topological and fractal dimensions, b) Regular self-similar entities with a fractal dimension greater than their topological one and c) Irregular self-similar entities with a fractal dimension much greater than their topological one. The usual geometrical entities lie on the first group: zero dimensional point, one dimensional line, two dimensional plane surface, and so. The most interesting ones are the two last groups where the attractors and strange attractors lie.

However in Applied Physics it is no usual dealing with perfect mathematical entities. The mathematical models of the Fractal Geometry applied on real world physical processes have four characteristics (Guzmán et al. 1993; Pacheco 1994): a) They are difficult to compute because they need massive computation, b) It is a method for classification and not for prediction, c) The role of the Geometry depends on the range of values of the physical observable and its temporal evolution and d) They use to present several temporal/spatial scales imbedded in the same record. Undoubtedly the proper reading of the result will depend on the physics of the phenomenon, on its formulation in terms of fractals and on the physical intuition of the researcher.

Although the basic applications of fractals were developed to treat two dimensional objects (images) by means of the simple box-counting method (García & Otárola 1994), it is possible the computation of some fractal measures for one dimensional time series. In any case, and because in the case of the tides, there are several time scales embedded in the data, it is more proper to use the term of multifractal instead fractal. Perhaps this is the most remarkable geometrical property of the nature.

There is only one reference about the application of multifractal on tidal records. Alonso (1998) analyzed two high precision earth tides with a noise level of pico-gals from two superconducting gravity meters placed at Brussels (Royal Observatoire de Belgique) and in the Black Forest Observatory (Germany). A new parameter named stability of the tidal station was defined giving an additional parameter for classifying the earth tides gravity stations. One of his main conclusions was that very good quality time series, easily to predict by means of harmonic analysis, can be fractally very different. Being possible to scope very deep into the geometrical structure of the time series and determine which one has the best quality. Because fractals are extremely powerful tools for classification, it is possible to conclude from the numerical results that the station under study can present problems, even if the results of other classical procedures (i.e. the harmonic analysis) militate against the fractal conclusions. This was fairly demonstrated in Alonso (1998) for earth tides and now the study is extended on ocean tides.

The work has been organized as follows. The simple mathematical apparatus of fractal dimensions is presented in Section 2. The proper modification of some fractal concepts needed to apply the theory on tides is presented in the same section. Section 3 is devoted to describe the numerical experiments and to discuss the results. Finally the conclusions are drawn in section 4.

**FRACTAL CONCEPTS AND THEIR APPLICATION ON TIDAL RECORDS**

There are two main concepts in the large body of the Fractal Theory that are necessary to discuss and explain in order to apply on tides. Firstly it is the concept of dimension. This can be defined in many ways but only few definitions could be valid in the
frame that will be stated. The second one is the concept of self-similarity or scale invariant. This is more difficult to establish but it is the basis for the application of the Geometric Theory of the Measure applied on the Mean Sea Level (MSL) oscillations.

1. Dimensions

From natural the dimension of a vector or functional space is the minimum number of coordinates needed to locate any point in that space. For the real physical world there are zero dimensional points, one dimensional lines, two dimensional plane surfaces, three dimensional objects and the four dimensional Minkowski space-time. In addition it is possible to choose any proper system of coordinates to study a given problem. Changing the system of coordinates can be understood as a distortion of the body under study or a distortion of the same space keeping the dimensions of the problem.

The computation of the dimension for simple and common geometrical objects has no problem (Guzmán et al. 1983). However there are many geometrical objects with a priori unknown dimension. As an example it is possible to point out the attractors in the phase space of a given dynamical system. The dynamical systems can have several attractors in the phase space, each one with their own dimension. Hence it is necessary to adopt other methods to estimate the dimension of geometrical objects.

The estimation of the $D_q$ dimension of an attractor in the phase space generated by a time series is computed easily by means of temporal differences and by the Takens' theorem (Takens 1981). The general expression was due to (Grassberger & Procaccia 1984):

$$ D_q = \lim_{\varepsilon \to 0} \frac{\log(C_q(\varepsilon))}{\log(\varepsilon)} \quad (1) $$

Being $\varepsilon$ the size step and $C_q(\varepsilon)$ the generalized integral of correlation defined as:

$$ C_q(\varepsilon) = \left[ \frac{1}{M^q} \sum_{j=1}^{M^q} \left( \frac{1}{M} \sum_{i=1}^{M} \theta(\varepsilon - |y_i - y_j|) \right)^{-q} \right]^{\frac{1}{q-1}} \quad (2) $$

Where $q$ is the type of dimension, $M$ is the so called embedding dimension, $y_k$ is the k-thm orbit used in the building of the Takens' matrix and $\theta(\cdot)$ is the step or Heaviside function. A nice and simple introduction to the problem and to its proper reading can be found in Broomhead & King (1985). It is possible to compute the dimensions for any arbitrary integer value of $q$, but the reading with $q > 2$ is not clear and nor stated.

In the fractal problems there are three types of dimensions. With $q=0$ the Hausdorff dimension is obtained, $D_0$, and it is the same that obtained by the box-counting method (Guzmán et al. 1983). The problem with the application of $D_0$ lies in that it does not take into account the density of points, giving the same weight to boxes with one point or full of them.
With $q=1$ the information dimension is computed, $D_1$, that introduces the correction for the density of points. However it does not take into account the correlation between orbits. In order to deal with time series the best choice is $q=2$ and obtain the correlation dimension, $D_2$. This takes into account the correction of the density of points and the correlation between orbits, giving an unbiased value of dimension except by the same length of the data set. Because we are dealing with long time series it is possible to consider that the resulting estimations will be unbiased. However the authors will present the results for the three described dimensions and their consequences.

Independently of the value of $q$, some considerations must be done. A typical example of the plot of $C_q(\varepsilon)$ vs. $\varepsilon$, in linear scale, is presented in Figure 1. It is clear than as a function of the length of the data set, the range of $\varepsilon$ and the physics of the process; the linear regression can be carried out with higher statistical confidence. The value of the q-dimension will depend of where the regression is performed in Figure 1. The segment number 3 corresponds to very high values of $\varepsilon$ and the segment number 1 to very small values. Only the segment 2 will provide valuable information about the geometrical properties of the set and it corresponds to optimum values of the size step. Region 2 is usually called scaled region. In practice the regions 1 and 3 use to be very small, however the starting and ending points of the regression depend on the experience of the researcher.

2. Self-similarity of tidal records

The tide is the observed result of a multidimensional dynamical system. The number of parameters needed to specify the location of the free surface in the time $t$ is $3N+1$, being $N$ the number of resolved waves taking into account the amplitudes, phase lags and frequencies. So the working space is a $3N+1$ parametric one. All of them are resumed in the Fourier decomposition of the vertical tide:

$$\eta(t) = z_0 + \sum_{i=1}^{\infty} A_i \cdot \cos(\omega_i \cdot t + \phi_i)$$  \hspace{1cm} (3)

Where $\eta(t)$ is the location of the free surface respect to the mean sea level $z_0$, $A_i$ is the amplitude, $\omega_i$ is the frequency and $\phi_i$ is the phase lag. The infinity must be understood as all the waves that can be resolved. Such number will depend on the length of the record and on the Rayleigh's criterion of the harmonic analysis. The Eq. 3 describes the projection of the phase space in the vertical of a forced oscillator around an equilibrium point, the mean sea level. That point, if it is stable (no motion) must have a null geometrical dimension and it would be a common geometrical object. Because the tide is composed by many embedded time scales, the tide is a multifractal. The scheme of the dimensions involved in the problem of the vertical tide is presented in Figure 2. It is possible
to define a two dimensional local mean sea level surface, the direction of the projection is one dimensional, the position of the free surface is determined with a $3N+1$ dimension and the attractor of the system, the mean sea level, has an unknown dimension that must be estimated.

The attractor is characterized by the temporal derivative of the equation of motion in a small segment around itself (Guzmán et al. 1993). For the case of tidal records the solution of the problem is easy because it is usual to assume that Eq (3) reproduce quite well the experimental observations. Denoting by $y'$ to the temporal derivative of $\eta(t)$, the attractor can be classified as presented in Table 1 as super-attractive, attractive, indifferent and repulsive. This works quite well if no instrumental or environmental noises are acting or with analytical functions as the Tide Generating Potential (Alonso 1998). In addition the mathematical decomposition of Eq (3) is not perfect in any case when dealing with tides because the number of resolved waves is much smaller that the existing ones. Hence the classification of Table 1 must be considered with an unknown error in the computation of the derivatives (Alonso 1998).

Table 1. Classification of attractors.

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Denomination</th>
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<tbody>
<tr>
<td>$y'=0$</td>
<td>Super-attractive</td>
</tr>
<tr>
<td>$y'&gt;(0,1)$</td>
<td>Attractive</td>
</tr>
<tr>
<td>$</td>
<td>y'</td>
</tr>
<tr>
<td>$</td>
<td>y'</td>
</tr>
</tbody>
</table>

The Fractal Geometry gives extremely simple and powerful tools to evaluate the strength of the attractor that must be understood as the averaged numerical derivative of the equation of motion around the location of the attractor. Only in this way it is possible to estimate the rate of motion of the attractor or the mean sea level. Because of this the dimension of the attractor can be thought as a parameter to characterize the stability of the tidal station. The recovering of the phase space from the experimental data of the Ceuta tidal station is presented in Figure 3 for several values of embedding dimensions. If $m=24$ is considered, the attractor is well defined in the centre of the scatter plot. If the embedding dimension is higher, $m=336$ or 672, the dynamic of the attractor is occulted in the plots but the attractor is present, governing the dynamic in the phase space.

The concept that acts a key-stone in the fractal analysis of tidal records is the self-similarity or scale invariance. From the mathematical model of Eq 3, the tidal record must be considered self-similar. This is valid for infinity Fourier time series. However if this is truncated, as it is the case of tidal records, any change in the scale will act as a filter. Because of this the minimum and maximum values of the size step, $\varepsilon$, must be defined. It can not start with $\varepsilon\to 0$. Only in this way it is possible to adapt the self-similarity to records of any geophysical variable affected by tides.

The next problem is considering the multifractal property of the nature. Because the tide is a multifractal it is necessary to compute the dimension with several values of embedding dimension. In order to show this the typical tidal time scales have been considered: 6, 12, 24, 336 y 672 hours, corresponding

Table 2. Fractal dimensions for the Algeciras tidal station.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$D_0$</th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.4583</td>
<td>0.4029</td>
<td>0.4442</td>
</tr>
<tr>
<td>12</td>
<td>0.0309</td>
<td>0.5727</td>
<td>0.6017</td>
</tr>
<tr>
<td>24</td>
<td>0.7136</td>
<td>0.8739</td>
<td>1.2973</td>
</tr>
<tr>
<td>336</td>
<td>0.6697</td>
<td>1.1543</td>
<td>0.1846</td>
</tr>
<tr>
<td>672</td>
<td>0.5525</td>
<td>1.0722</td>
<td>0.0693</td>
</tr>
</tbody>
</table>
to fourth diurnal, semidiurnal, diurnal, fortnightly and monthly tidal waves. In addition it must be pointed out that the input data for fractal analysis must be pikes free and non detided. The main reason for the first is clear and for the second is that the driving force is eliminated and the attractor can not be observed (Alonso 1998).

NUMERICAL EXPERIMENTS AND DISCUSSION

Three tidal stations have been considered: Algeciras, Ceuta and Malaga. The time series passed the usual quality controls in order to correct pikes and instrumental errors. The available time span is detailed in Tables 2 to 4. The analyzed tidal stations have the typical mid-latitude semidiurnal periodicities. Although only the D2 fractal dimension must be computed (section 2), the three dimensions \((q=0,1,2)\) will be considered in order to compare the results and to show that visual inspected well behavioural tidal records can be very different and with low quality being better do not use them. The dimensions were estimated by the use of Eq (2) with a linear regression from Eq (1) selecting \(q=0,1,2\) consecutively. Results are presented in Tables 2 to 4. Due to the multifractal property of the tide five embedding dimensions were selected from the physics of tides: 6, 12, 24, 336 and 672 hours. If several years long time series were available, the embedding dimension could take the values corresponding to six month or one year as showed in Alonso (1998).

The theoretical reference value has been taken from Alonso (1998) as the fractal dimension of a synthetic time series of 21 years long from the RATGP95 tide generating potential (Roosbeek 1985). Such a value is \(5\times10^{-6}\) mgals/year, practically zero, as it corresponds to a quasi infinity Fourier time series. This conclusive value points out that if a time series is perfectly harmonic, the fractal dimension of the attractor must tend to zero. If the obtained value is higher there is some undetermined problem with the time series. It is worth worthy hoping that the fractal dimension of ocean tidal records be so small than for earth tides (Alonso 1998) because the water level recorders do not suffer the permanent and exhaustive quality controls of the superconducting gravity meters.

For the Algeciras tidal station a quite constant value of fractal dimension is observed for \(m=6\) close to 4 mm/year. For the main periodicity, \(m=12\), the trend or fractal dimension is over 6 mm/year. The value of \(D_2\) is quite high for \(m=24\), 1.29 mm/year, and \(D_1\) for \(m=336\) and \(m=672\) with more than 1cm/year. However they do not must be taken into account because the diurnal waves are very weak. The value of \(D_2\) is quite small for the rest of embedding dimensions. It must be pointed out that all the computed values are quite different, even in their magnitude. This is no normal at all and implies that there must be some problems with this time series with the presence of some undesirable phenomena.

The values of the fractal dimensions for Ceuta and Malaga tidal stations are perfect. The mean sea level does not have practically motion and the fractal dimensions are one order of magnitude lesser than the computed for Algeciras tidal station. The results are quite homogeneous with the same order of magnitude also. The exception, pointed out above, is the value of \(D_0\) for the embedding dimension of 24 in the Malaga

| Table 3. Fractal dimensions for the Ceuta tidal station. |
|-----------------|--------|--------|--------|
| **Ceuta 01/01/1997-01/01/1998** |
| \(m\) | \(D_0\) | \(D_1\) | \(D_2\) |
| 6   | 0.6184 | 0.0245 | 0.0246 |
| 12  | 0.6245 | 0.1520 | 0.0412 |
| 24  | 0.6097 | 0.0369 | 0.0414 |
| 336 | 0.5816 | 0.0579 | 0.0661 |
| 672 | 0.6472 | 0.0596 | 0.0541 |
tidal station. This can be neglected from the analysis without problem because the diurnal periodicity has not importance in that place.

When comparing all the computed values for the three tidal stations the results for Algeciras are the anomalous among them. They are one order of magnitude greater. This does not mean that the mean sea level is rising at the north coast of the Strait of Gibraltar while at the south are very stable. However they imply the presence of instrumental effects that recommend do not use the time series for the analysis of the mean sea level. It is necessary to make a more exhaustive control of the Algeciras tidal station.

These results point out that the tidal records at Ceuta and Malaga can be used to study the mean sea level while the tidal record at Algeciras must be put in doubt seriously.

CONCLUSIONS

The authors have computed the fractal dimensions of three tidal stations of the South of Spain. The Fractal Geometry allows the contraction of the dynamic of the multidimensional dynamical system of tides around a single attractor point identified on the Mean Sea Level of the analyzed time series. Among the three well known fractal dimensions (Hausdorff, information and correlation dimensions) the most proper one to be applied on time series with organized motion is the $D_2$ dimension.

From the numerical results the Algeciras tidal station presents any unknown kind of problem (i.e. from maintenance related to the clock of the water level recorder, calibration, etc) that must be deeply implying that the data records in such place must not be used for the determination of the Mean Sea Level in the area.

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